

# CLASS: X

## MATHEMATICS

<b>1. REAL NUMBERS :</b>	<b>06 MARKS</b>
<b>2. PAIR OF LINEAR EQUATIONS IN TWO VARIABLES</b>	<b>12 MARKS</b>
<b>3. QUADRATIC EQUATIONS</b>	
<b>4. ARTHMETIC PROGRESSION</b>	
<b>5. COORDINATE GEOMETRY</b>	
<b>6. GEOMETRY (I) TRIANGLES</b>	<b>08 MARKS</b>
<b>(II) CIRCLE</b>	
<b>(III) CONSTRUCTION</b>	
<b>7. TRIGONOMETRY</b>	<b>06 MARKS</b>
<b>8. MENSURATION</b>	<b>06 MARKS</b>
<b>(I) AREAS RELATED TO CIRCLES</b>	
<b>(II) SURFACE AREA AND VOLUMES</b>	<b>04 MARKS</b>
<b>9. STATISTICS</b>	
<b>TOTAL :</b>	<b>48 MARKS</b>

# 1. REAL NUMBERS

*TOTAL MARKS 06 (1 VSA + 1 SA1 + 1 SA2)*

GIST OF THE LESSON :-

- EUCLID'S DIVISION LEMMA AND ALGORITHM
- FUNDAMENTAL THEOREM OF ARITHMETIC
- REVISITING IRRATIONAL NUMBERS
- REVISITING RATIONAL NUMBERS AND THEIR DECIMAL EXPANSION

**VOCABULARY---** 1. TERMINATING DECIMALS= ENDING DECIMALS (ON DIVISION, REMAINDER COMES OUT ZERO AFTER SOME STEPS . E.G.  $23/5 = 4.6$  AND  $21/7 = 3$ )

2. NON TERMINATING DECIMALS = NEVER ENDING (WHEN ON DIVISION REMAINDER NEVER COMES ZERO AND DIVISION CONTINUES. E.G.  $10/3 = 3.33333\ldots$  AND  $22/7 = 3.142857\ldots$ )

3. RECURRING = REPEATING E.G.  $10/3 = 3.33333\ldots$

## CONCEPT 1 ----- EUCLID'S DIVISION LEMMA AND ALGORITHM

1. State Euclid's division lemma.
2. Find HCF of (i) 25 and 70 (ii) 120 and 72 (iii) 1288 and 575 using Euclid's division lemma.
3. Divide (i) 48 by 6 (ii) 125 by 41 and write in the form of Euclid's division lemma.
4. Show 107 in the form of  $4q + 3$  for some positive integer q.
5. Using EDL show that the square of any positive integer is either of the form  $3m$  or  $3m + 1$  for some integer m.
6. Show that any positive odd integer is of the form  $4q + 1$  and  $4q + 3$  where q is any positive integer.

## CONCEPT 2 ----- FUNDAMENTAL THEOREM OF ARITHMETIC

1. State Fundamental theorem of arithmetic .
2. What are Prime and Composite Numbers.
3. Write all Prime numbers between 1 and 50.
4. Write prime Factorisation of (i) 156 (ii) 660 (iii) 816.
5. Find LCM and HCF of (I) 48 and 128 (ii) 33 and 121 (iii) 288 and 120 using Fundamental theorem of arithmetic.
6. Find LCM and HCF of 26 and 91 and verify the formula :-  $LCM(a,b) \times HCF(a,b) = a \times b$ .
7. Check whether  $5^n$  ends with 0 for any natural number n?
8. Check whether  $5 \times 3 \times 11 + 11$  and  $13 \times 5 \times 3 \times 2 + 3$  are composite or not

## CONCEPT 3 ----- REVISITING IRRATIONAL NUMBERS

1. Write any five rational and irrational numbers.
2. Which of following are rational?  
(i)  $\frac{3}{4}$  (ii)  $5 - \sqrt{2}$  (iii)  $(4 + \sqrt{3})(4 - \sqrt{3})$  (iv)  $10 / 2\sqrt{5}$  (v)  $7 - \sqrt{4}$
3. Show that  $\sqrt{3}$  is an irrational number.

4. Show that  $6 + 3\sqrt{2}$  is an irrational number.
5. Show that  $\sqrt{50}$  is an irrational number.

#### CONCEPT 4----- REVISITING RATIONAL NUMBERS AND THEIR DECIMAL EXPANSION

1. Which of the following decimal expansions are rational ?  
 (i) 0.30130013000100120..... (ii) 110.047 (iii) 34.2222222.....  
 (iv) 29.616161.....
2. What is the condition on p and q that the decimal expansion of a rational number  $p/q$  ( $q \neq 0$ ) is terminating.
3. Without actual division state which of the following decimal expansions have a terminating decimal or non-terminating recurring.  
 (i)  $7/100$  (ii)  $13/425$  (iii)  $23/55$  (iv)  $45/5^2 \times 4 \times 3$  (v)  $17/9$

## 2. Pair Of Linear Equations in Two Variables

### IMPORTANT CONCEPTS

- 1 **Linear equation in two variables:** An equation in the form  $ax + by + c = 0$ , where a, b and c are real numbers, and a and b are not both zero ( $a^2 + b^2 \neq 0$ ), is called a linear equation in two variables x and y. e.g.  $3x - y + 7 = 0$ ,  $7x + y = 3$
- 2 **Solution of a linear equation in two variables:** Any pair of values of x and y which satisfies the equation  $ax + by + c = 0$ ;  $a \neq 0$ ,  $b \neq 0$  is called a solution of a linear equation.
- 3 Every solution of the equation is a point on the line representing it.
- 4 Each solution (x, y) of a linear equation in two variables,  $ax + by + c = 0$ , corresponds to a point on the line representing the equation, and vice versa.
- 5 **General form of pair of linear equations in two variables:**  
 The general form for a pair of linear equations in two variables x and y is  
 $a_1x + b_1y + c_1 = 0$   
 $a_2x + b_2y + c_2 = 0$ ,  
 Where  $a_1, b_1, c_1, a_2, b_2, c_2$  are all real numbers and  $a_1^2 + b_1^2 \neq 0$ ,  $a_2^2 + b_2^2 \neq 0$ .

## 6 Geometrical representation of pair of linear equations in two variables

The geometrical representation of a linear equation in two variables is a straight line.

## 7 Methods of solving pair of linear equations in two variables

A pair of linear equations can be solved by the following methods:

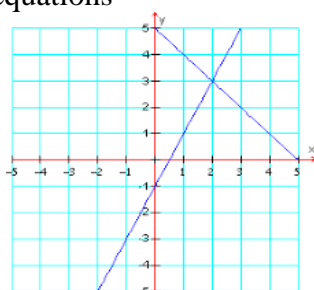
Graphical method

Algebraic method

## 8 Graphical Method:

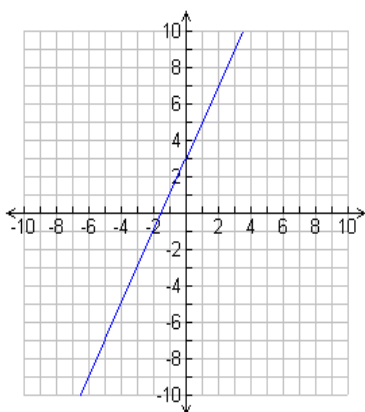
For a given pair of linear equations in two variables, the graph is represented by two lines.

- 1) If the lines intersect at a point, that point gives the unique solution for the two equations. If there is a unique solution of the given pair of equations, the equations are called consistent pair of equations



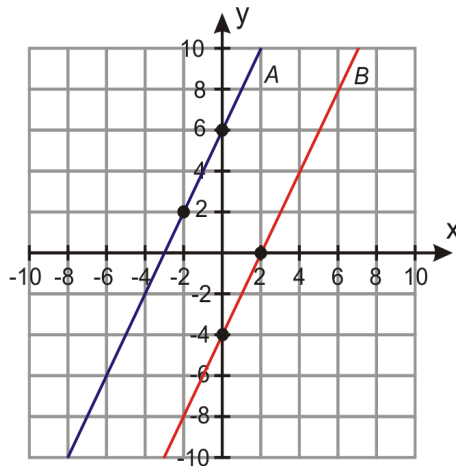
a.

- 2) If the lines coincide, there are infinitely many solutions for the pair of linear equations. In this case, each point on the line is a solution. If there are infinitely many solutions of the given pair of linear equations, the equations are called consistent.



1.

- 3) If the lines are parallel, there is no solution for the pair of linear equations. If there is no solution of the given pair of linear equations, the equations are called inconsistent.



### 9. Algebraic method of solving pair of linear equations in two variables

There are three methods for finding the solutions of the pair of linear equations algebraically

- 1) Substitution method
- 2) Elimination method
- 3) Cross-multiplication method

### 10. Conditions for solvability of pair of linear equations in two variables

The pair of linear equation

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

- 1) Has a unique solutions if

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

a. .

- 2) Has infinitely many solutions if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

a.

- 3) has no solution if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

a.

#### KEY WORDS OF THE CHAPTER

1. **Geometrical representation** : Solving the equations graphically .
2. **Unique solution** : Pair of equations one and only one solution.
3. **Consistent** : Pair of equations having one or more solutions.
4. **Coincide** : Overlapping.
5. **Inconsistent** : Pair of equations having no solution.
6. **Intersecting Lines**: When two lines meet each other at one point.

**7. Parallel lines : When two lines do not meet each other at any point however far they are extended .**

**QUESTIONS**

1. Show that  $x = 2$  ,  $y = -2$  is a solution of the linear equation  $5x + 3y = 4$ .
2. Show that  $x = 7$  ,  $y = -2$  is a solution of the linear equation  $3x + 2y = 17$ .
3. Find the value of  $k$  if  $x = 3$  and  $y = 2$  is a solution of the equation  $4x + (k - 1) y = 16$ .
4. Draw the graph of the  $2x - 3y = 5$  .
5. Draw the graph of the equation  $3x + 4y = 11$ .
6. On comparing the ratio  $\frac{a_1}{a_2}, \frac{b_1}{b_2}, \frac{c_1}{c_2}$  , find out whether the lines representing the following linear equations intersect at a point are parallel or coincide.
  - a.  $3x - 5y + 6 = 0$   
 $7x + 6y - 9 = 0$
  - b.  $9x - 3y + 12 = 0$   
 $18 + 6y + 2 = 0$
  - c.  $6x - 3y + 10 = 0$   
 $2x - y + 9 = 0$
7. For each of the following system of equations determine the value of  $k$  for which the given system of equations has a unique solution
  - a.  $x - 2y = 2$
  - b.  $3x + ky = -5$
8. For what value of  $k$  , will the following system of equations have infinitely many solutions
  - a.  $2x + 3y = 4$
  - b.  $(k + 2)x + 6y = 3k + 2$
9. Solve graphically the following system of equations
  - a.  $x + 2y = 5$   
 $3x + 6y = 15$
  - b.  $x + 3y = 6$   
 $2x - 3y = 12$
  - c.  $x + 3y = 6$   
 $2x - 3y = 12$
  - d.  $3x + y = 8$   
 $6x + 2y = 6$
10. Solve the following pair of linear equations
  - a.  $x + y = 14$   
 $x - y = 4$
  - b.  $3x + 4y = 10$   
 $2x - 2y = 2$

c.  $3x - 5y = 4$   
 $9x = 2y + 7$

11. Seven times a two digit number is equal to four times the number obtained by reversing the order of its digits. If the difference of the digits is 3, determine the number.
12. Five years hence, the age of Jacob will be three times that of his son. Five years ago, Jacob's age was seven times that of his son. What are their present ages?
13. Five years ago, Salma was thrice as old as Sonu. Ten years later, Salma will be twice as old as Sonu. How old are Nuri and Sonu?
14. A fraction becomes  $\frac{1}{3}$  when 1 is subtracted from the numerator and it becomes  $\frac{1}{4}$  when 8 is added to its denominator. Find the fraction
15. A boat goes 30 km upstream and 44 km downstream in 10 hours. In 13 hours, it can go 40 km upstream and 55 km down-stream. Determine the speed of the stream and that of the boat in still water.
16. Solve the following pairs of equations by reducing them to a pair of linear equations:

a)  $\frac{1}{2x} + \frac{1}{3y} = 2$

$$\frac{1}{3x} + \frac{1}{2y} = \frac{13}{6}$$

b)  $\frac{5}{x-1} + \frac{1}{y-2} = 2$

$$\frac{6}{x-1} - \frac{3}{y-2} = 1$$

## ANSWER KEY

Ans 3. 3

Ans 6 a. Intersecting Lines

b Coincident lines

c Parallel Lines

Ans7  $k \neq -\frac{2}{3}$

Ans 8  $k = 2$

Ans 9 Infinitely Many Solutions

a. Unique Solution

b. No Solution

Ans 10  $x = 9, y = 5$

a.  $x = 2, y = 1$

b.  $x = \frac{9}{13}, y = -\frac{5}{13}$

c.  $x = -21, y = 17$

Ans 11 36

Ans 12 Age of Jacob = 40 Years, Age of Son = 10 Years,

Ans 13 Age of Salma = 50 Years, Age of Sonu = 20 Years,

Ans 14 Fraction =  $\frac{5}{12}$

Ans 15 Speed in Still water = 8 Km/hr, Speed of Stream = 3 Km/hr

Ans 16 a)  $x = \frac{1}{2}, y = \frac{1}{3}$

b)  $x = 4, y = 5$

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### 3. Quadratic Equations

#### Gist of the lesson

1. A quadratic equation in the variable  $x$  is of the form  $ax^2 + bx + c = 0$ , where  $a, b, c$  are real numbers and  $a \neq 0$ .
2. A real number  $\alpha$  is said to be a root of the quadratic equation  $ax^2 + bx + c = 0$ , if  $a\alpha^2 + b\alpha + c = 0$ . The zeroes of the quadratic polynomial  $ax^2 + bx + c$  and the roots of the quadratic equation  $ax^2 + bx + c = 0$  are the same.
3. If we can factorise  $ax^2 + bx + c, a \neq 0$ , into a product of two linear factors, then the roots of the quadratic equation  $ax^2 + bx + c = 0$  can be found by equating each factor to zero.
4. A quadratic equation can also be solved by the method of completing the square.
5. Quadratic formula: The roots of a quadratic equation  $ax^2 + bx + c = 0$  are given by  $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ , provided  $b^2 - 4ac \geq 0$ .
6. A quadratic equation  $ax^2 + bx + c = 0$  has
  - (i) two distinct real roots, if  $b^2 - 4ac > 0$ ,
  - (ii) two equal roots (i.e., coincident roots), if  $b^2 - 4ac = 0$ , and
  - (iii) no real roots, if  $b^2 - 4ac < 0$ .

#### Level 1

1. Find  $k$  for which the sum & product of the roots of quadratic equation  $kx^2 - 4x + 4k = 0$  are equal.
2. Find the value of  $K$  for which  $x^2 - (K + 1)x + 4 = 0$  has equal roots.
3. Factorize :  $x^2 - x - 12 = 0$
4. Is  $x = -3$  a solution of the equation  $x^2 - 3x + 7 = 0$  ?
5. Show that  $x = -2$  is a solution of the equation  $2x^2 - 4x - 16 = 0$
6. Find the discriminate of the quadratic equation  $2\sqrt{5}x^2 + 8x + \sqrt{5} = 0$
7. Write the nature of roots of quadratic equation  $3x^2 + 4\sqrt{3}x + 6 = 0$
8. For what value of  $k$ , the quadratic equation  $5x^2 - kx + 4$  have equal roots.



9. For what value of P, the quadratic equation  $px(x - 4) + 64 = 6$  has equal roots  $p \neq 0$ .
10. Find the roots of equation  $5\sqrt{2}x^2 + 7x + \sqrt{2} = 0$

### Level 2

- For what value of k, the quadratic equation  $kx^2 - 2\sqrt{3}kx + 9 = 0$  has equal roots?
- Find K for which the roots of quadratic equation are  $(K - 9)x^2 + 2(K - 9)x + 4 = 0$  equal.
- Find the sum and products of the roots of equation  $\sqrt{3}x^2 + 9x + 6\sqrt{3} = 0$
- Find two consecutive odd positive integers whose squares have sum 290.
- If 2 is a root of the equation  $ax^2 + ax + 2 = 0$  and  $x^2 - x - b = 0$  then find a/b.
- For what value of k the quadratic equation has equal roots? Hence find the roots of the equation,  $2kx^2 - 4(k - 2)x + 9$
- For what value of k, the equation  $x^2 + (k + 1)x + (k + 4) = 0$  has equal roots?
- If  $x^2 + 5kx + 16 = 0$  has no real roots, then
  - $k > \frac{8}{5}$
  - $k > -\frac{8}{5}$
  - $-\frac{8}{5} < k < \frac{8}{5}$
  - $0 < k < \frac{8}{5}$
- Solve for x :  $2\sqrt{3}x^2 - 2\sqrt{2}x - \sqrt{3} = 0$
- For what value of k, does the quadratic equation  $(k - 3)x^2 + 2(k - 3)x + 5 = 0$  have equal roots.
- Solve for x :  $3\sqrt{2}x^2 - 13x + 5\sqrt{2} = 0$

### Level 3

- Find the roots of  $4x^2 - 4a^2x + a^4 - b^4 = 0$
- Solve for X :  $4x^2 + 8bx - (4a^2 - 4b^2) = 0$
- Find the roots of the following equation.
 
$$\frac{1}{x+3} - \frac{1}{x+4} = \frac{1}{42}$$
- Solve  $\left(\frac{2x+5}{x-5}\right) - 2\left(\frac{x-5}{2x+5}\right) = \frac{-17}{28}$
- A car left 30 minutes later than the scheduled time. In order to reach its destination 150 km away in time, it has to increase its speed by 25 km/hr from its usual speed. Find its usual speed.
- The speed of a boat in still water is 15 km/hr. It goes 30 km upstream and returns downstream to the original point in 4 hrs 30 minutes. Find the speed of the stream.
- Three years ago Atul's age was four times Parul's age. After 5 years from now, Atul's age will be twice Parul's age. Find their present ages.
- The perimeter of a rectangle is 90 cm and its area is  $486 \text{ cm}^2$ . Find the dimensions of the rectangle.
- Sum of two numbers is 27. The sum of their reciprocals is  $\frac{3}{20}$ . Find the numbers.
- A two digit number is such that the product of its digits is 21. If 36 is subtracted

from the number the digits are interchanged. Find the number.

11. Solve for  $x : 5\left(\frac{3x+5}{x-3}\right) - 33\left(\frac{x-3}{3x+5}\right) = -52$

12. The difference of areas of two squares is  $549 \text{ cm}^2$ . If the difference of their perimeters is 12 cm find the sides.

13. Find the quadratic equation having roots  $(1 + \sqrt{2})$  and  $(1 - \sqrt{2})$ .

14. Form the quadratic equation whose roots are  $7 + \sqrt{3}$  and  $7 - \sqrt{3}$ .

## 4. ARITHMETIC PROGRESSION

Numbers can have interesting patterns.

Here we list the most common patterns and how they are made.

### Arithmetic Sequences

An Arithmetic Sequence is made by **adding** some value each time.

**Example:**

1, 4, 7, 10, 13, 16, 19, 22, 25, ...

This sequence has a difference of 3 between each number.

The pattern is continued by **adding 3** to the last number each time.

**Example:**

3, 8, 13, 18, 23, 28, 33, 38, ...

This sequence has a difference of 5 between each number.

The pattern is continued by **adding 5** to the last number each time.

The value added each time is called the "common difference"

What is the common difference in this example?

19, 27, 35, 43, ...

Answer: The common difference is **8**

The common difference could also be negative, like this:

25, 23, 21, 19, 17, 15, 13, 11, 9, ...

This common difference is **-2**

The pattern is continued by **subtracting 2** each time.

## Arithmetic progression

In Mathematics, an **arithmetic progression** (AP) or **arithmetic sequence** is a sequence of numbers such that the difference between the consecutive terms is constant. For instance, the sequence 3, 5, 7, 9, 11, 13, ... is an arithmetic progression with common difference of 2.

If the initial term of an arithmetic progression is  $a_1$  and the common difference of successive members is  $d$ , then the  $n$ th term of the sequence is given by a

### Level 1

#### CONCEPTS IN ARITHMETIC PROGRESSION

##### 1. What do mean by an AP.

**Ans-1** The full form of AP (ARITHMETIC PROGRESSION) An AP is like,

$$a, a+d, a+2d, a+3d, a+4d, \dots$$

where,  $a$  = first term and  $d$  = common difference

example: 2, 5, 8, 11, 14

here,  $a=2$  and  $d=3$

##### 2. How to derive an $n^{\text{th}}$ of an AP.

**Ans -2**  $n^{\text{th}}$  of an AP =  $a_n = a + (n-1)d$

where,  $a$  = first term

$n$  = number of terms

$d$  = common difference

example: find the 10<sup>th</sup> term of an AP 2, 4, 6, 8, .....

answer:  $a = 2$ ,  $n = 10$ ,  $d = 2$

$$a_{10} = 2 + (10-1)2$$

$$a_{10} = 2 + (9)2$$

$$a_{10} = 2 + 18$$

$$a_{10} = 20$$

therefore, 10<sup>th</sup> term of an AP is 20.

#### LEVEL-1 ( Questions Based on $n^{\text{th}}$ terms)

1. Find the 5th term of AP: 1, 5, 9, .....
2. Find the 10th term of AP: 5, 10, 15, .....
3. Find the 20th term of AP: 1, 4, 7, .....
4. Find the 30th term of AP: 3, 6, 9, .....
5. Find the 50th term of AP: 2, 5, 8, .....

#### LEVEL-2 (Questions)

1. Write the first four terms of the when the first terms and common difference are given.
  - a.  $a = 10$ ,  $d = 10$
  - b.  $a = -2$ ,  $d = 0$
2. Write the first terms and common difference of given AP:
  - a. -10, -6, -2, 2, ....
  - b. 0, -4, -8, -12, ....

3. Find whether these are AP or not
  - a. 2,3,4,6,7,9.....
  - b. 1,2,3,4,4.....
  - c. 2,4,6,8.....
4. Find the next term of the AP: 3,7,11,15.....  
Find 10<sup>th</sup> term of AP: 2,4,6,8,10.....,28.

### **LEVEL-3 (Questions)**

1. Find the of all three digits natural numbers which are divisible by 9.
2. Find the first term if  $a_5 - a_3 = 4$ .
3. Find the value of k if  $8k+4$ ,  $6k-2$ ,  $2k-7$  are three consecutive terms of an AP.
4. Find the value of k if  $(2k+1)$ , 8,  $3k$  are in AP.
5. Find the common difference if AP is  $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32} \dots$
6. An AP consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29th term.
7. Which term of the AP: 3, 15, 27, 39, ... will be 132 more than its 54th term?
8. . How many multiples of 4 lie between 10 and 250?

### **Sum of first n terms in an arithmetic progression**

$$S_n = n/2 [ 2a + (n-1)d ]$$

where a = the first term,

d= common difference,

$$l = t_n = n^{\text{th}} \text{ term} = a + (n-1)d$$

### **LEVEL-1**

Find the sum of the following APs

- (i) 2, 7, 12, ....., to 10 terms
- (ii) -37, -33, -29, ..... to 12 terms
- (iii) 0.6, 1.7, 2.8, ..... to 100 terms

(IV) Find  $4 + 7 + 10 + 13 + 16 + \dots$  up to 20 terms

### **LEVEL-2**

Q1 Find  $6 + 9 + 12 + \dots + 30$

Q2  $-5 + (-8) + (-11) + \dots + (-230)$

Q3 . In an AP Given  $a = 5$ ,  $d = 3$ ,  $a_n = 50$ , find n and  $S_n$ .

Q4 Given  $a_3 = 15$ ,  $S_{10} = 125$ , find d and  $a_{10}$

Q5 Given  $d = 5$ ,  $S_9 = 75$ , find a and  $a_9$ .

### LEVEL-3

**Q1.** How many terms of the AP: 9, 17, 25,... Must be taken to give a sum of 636?

**Q2** The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.

**Q3** The first and the last term of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is the sum?

**Q4.** Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively

**Q5** If the sum of the first  $n$  terms of an AP is  $4n - n^2$ , what is the first term (that is  $S_1$ )? What is the sum of first two terms? What is the second term? Similarly, find the 3rd, the 10th and the  $n$ th term.

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## **5. GEOMETRY**

### **TOPIC: TRIANGLES**

1. Theorem: if a line divides any two sides of a triangle in the same ratio then the line is parallel to the third side.
2. The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.
3. In a right triangle, the square of the hypotenuse is equal to the sum of the squares of other two sides.
4. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points the other two sides are divided in the ratio (**Basic Proportional Theorem OR Thales Theorem**)
5. PQR is a triangle right angled at P and M is a point on QR such that  $PM \perp QR$ , Show that  $PM^2 = QM \cdot MR$
6. A ladder 10 m long reaches a window 8 m above the ground. Find the distance of the foot of the ladder from base of the wall.
7. An aero plane leaves an airport and flies due north at a speed of 1000 km per hour, At the same time, another aero plane leaves the same airport and flies due west at speed of 1200 km per hour. How far apart will be the two planes after  $1\frac{1}{2}$  hours.
8. Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the square of its diagonals.
9. In a triangle if  $AD \perp BC$  Prove that  $AB^2 + CD^2 = BD^2 + AC^2$

10. In triangle PQR,  $ST \parallel QR$  if  $\frac{PS}{SQ} = \frac{PT}{TR}$  and  $\angle PST = \angle PRQ$  Prove that PQR is an isosceles triangle.
11. In triangle ABC.  $DE \parallel BC$ , if  $AD = 1.5\text{cm}$ ,  $AE = 1\text{cm}$  and  $DB = 3\text{cm}$  find EC
12. A girl of height 90cm is walking away from the base of a lamp-post at a speed of 1.2m/s if the lamp is 3.6m above the ground, find the length of her shadow after 4 seconds.
13. D is a point on the side BC of a triangle ABC, such that  $\angle ADC = \angle BAC$ , show that  $CA^2 = CB \cdot CD$
14. Let  $\triangle ABC \sim \triangle DEF$ , and then areas be respectively  $64\text{cm}^2$  and  $121\text{cm}^2$  if  $EF = 15.4\text{cm}$  find BC
15. Prove that the area of an equilateral triangle described on side of a square is equal to half the area of the equilateral triangle described on one of its diagonals.

## 6. COORDINATE GEOMETRY

### Important Concepts

1. In the ordered pair  $(p, q)$ ,  $p$  is called the *x-coordinate* or *abscissa* and  $q$  is known as *y-coordinate* or *ordinate* of the point.
2. The abscissa of a point is its perpendicular distance from y-axis. The ordinate of a point is its perpendicular distance from x-axis.
3. The abscissa of every point situated on the right side of y-axis is **positive** and the abscissa of every point situated on the left side of y-axis is **negative**.
4. The ordinate of every point situated above x-axis is **positive** and that of every point below x-axis is **negative**.
5. The ordinate of every point situated above x-axis is **positive** and that of every point below x-axis is **negative**.
6. The ordinate of every point situated above x-axis is **positive** and that of every point below x-axis is **negative**.
7. The abscissa of every point on y-axis is zero.
8. The ordinate of every point on x-axis is zero.
9. **DISTANCE FORMULA**

The distance between any two points  $P(X_1, Y_1)$  and  $Q(X_2, Y_2)$  is given by

$$PQ = \sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2}$$

**10.** In order to prove that a given figure is a :

- (i) Square, prove that four sides are equal and the diagonals are equal.
- (ii) Rhombus, prove that the four sides are equal.
- (iii) Rectangle, prove the opposite sides are equal and the diagonals are also equal.
- (iv) Parallelogram, prove that the opposite sides are equal.

(v) Parallelogram but not a rectangle, prove that its opposite sides are equal but diagonals are not equal.

**11.** Three points A, B and C are said to be collinear, if they lie on the same straight line.

**12.** For three points to be *collinear*, the sum of the distances between two pairs of points is equal to the third pair of points.

### 13. SECTION FORMULA

The coordinates of the point P(x, y) which divides the line segment joining A(x<sub>1</sub>, y<sub>1</sub>) and B(x<sub>2</sub>, y<sub>2</sub>) internally in the ratio m : n, are given by :

$$P(x, y) = \left( \frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right)$$

**14.** The coordinates of the mid-point M of a line segment AB with end points A(x<sub>1</sub>, y<sub>1</sub>) and B(x<sub>2</sub>, y<sub>2</sub>) are :

$$\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

**15.** The area of a  $\square ABC$  with vertices A(x<sub>1</sub>, y<sub>1</sub>) B(x<sub>2</sub>, y<sub>2</sub>) and C(x<sub>3</sub>, y<sub>3</sub>) is given by:

$$\text{Area} (\square ABC) = \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

**16.** Three given A(x<sub>1</sub>, y<sub>1</sub>) B(x<sub>2</sub>, y<sub>2</sub>) and C(x<sub>3</sub>, y<sub>3</sub>) are collinear if

$$\text{Area} (\square ABC) = 0$$

$$\text{i.e. } \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$$

### Questions :

**1.** Write Distance Formula.

**2.** Find the distance between the following pairs of points:

a. (2, 3), (4, 1) (ii) (-5, 7), (-1, 3)

**3.** Is the point (4, 4) equidistant from the points P(-1, 4) and Q(1, 0)?

**4.** Are the points A(4, 5), B(7, 6) and C(6, 3) collinear.

**5.** What is the distance of the point P (2, 3) from the x-axis?

**6.** What is the distance between A on the x-axis whose abscissa is 11 and B (7, 3)?

**7.** Show that the points (1, 7), (4, 2), (-1, -1) and (-4, 4) are the vertices of a square.

**8.** Check whether (5, -2), (6, 4) and (7, -2) are the vertices of an isosceles triangle.

**9.** Find the point on the x-axis which is equidistant from (2, -5) and (-2, 9).

**10.** Find a point on the y-axis which is equidistant from the points A(6, 5) and B(-4, 3).

**11.** Write section Formula.

**12.** In what ratio does the point (-4, 6) divide the line segment joining the points A(-6, 10) and B(3, -8)?

**13.** Find the ratio in which the y-axis divides the line segment joining the points (5, -6) and (-1, -4). Also find the point of intersection.

14. Find the ratio in which the line segment joining A(1, -5) and B(-4, 5) is divided by the  $x$ -axis. Also find the coordinates of the point of division.
15. If (1, 2), (4, y), (x, 6) and (3, 5) are the vertices of a parallelogram taken in order, find  $x$  and  $y$ .
16. Write the formula for calculating area of a  $\triangle ABC$  with vertices A( $x_1, y_1$ ) B( $x_2, y_2$ ) & C( $x_3, y_3$ ).
17. Find area of a  $\triangle$  whose vertices are (2, 3), (-1, 0), (2, -4).
18. Find the area of a triangle whose vertices are (1, -1), (-4, 6) and (-3, -5).
19. If A(-5, 7), B(-4, -5), C(-1, -6) and D(4, 5) are the vertices of a quadrilateral, find the area of the quadrilateral ABCD.
20. Find the value of  $k$  if the points (7, -2), (5, 1), (3,  $k$ ) are collinear.

## 7. Trigonometry

### Basic trigonometric identities

#### KEY CONTENTS:

The word 'trigonometry' is derived from the Greek words 'tri' (meaning three), 'gon' (meaning side) and 'metron' (meaning measure). In fact, trigonometry is the study of the relationships between the sides and the angles of triangle.

#### TRIGONOMETRIC RATIOS:

The trigonometric ratios of the angle A in triangle ABC are defined as follow:

$$\sin A = \frac{\text{side opposite to angle A}}{\text{hypotenuse}} = \frac{BC}{AC}$$

$$\cos A = \text{side adjacent to angle A} / \text{hypotenuse} = AB / AC$$

$$\tan A = \text{side opposite to angle A} / \text{side adjacent to angle A} = BC / AB$$

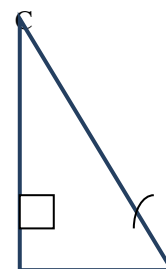
$$\operatorname{cosec} A = 1 / \sin \text{ of angle A} = \text{hypotenuse} / \text{side opposite to angle A} = AC / BC$$

$$\sec A = 1 / \cos \text{ of angle A} = \text{hypotenuse} / \text{side adjacent to angle A} = AC / AB$$

$$\cot A = 1 / \tan \text{ of angle A} = \text{side adjacent to angle A} / \text{side opposite to angle A} = AB / BC$$

#### TRIGONOMETRIC RATIOS OF SOME SPECIFIC ANGLES

Angle	0°	30°	45°	60°	90°
Sin A	0	1/2	1/√2	√3/2	1
Cos A	1	√3/2	1/√2	1/2	0
Tan A	0	1/√3	1	√3	Not defined
Cosec A	Not defined	2	√2	2/√3	1
Sec A	1	2/√3	√2	2	Not defined
Cot A	Not defined	√3	1	1/√3	0



A



## **TRIGONOMETRIC RATIOS OF COMPLEMENTRY ANGLES**

$$\sin(90^\circ - A) = \cos A$$

$$\cos(90^\circ - A) = \sin A$$

$$\tan(90^\circ - A) = \cot A$$

$$\cot(90^\circ - A) = \tan A$$

$$\sec(90^\circ - A) = \operatorname{cosec} A$$

$$\operatorname{cosec}(90^\circ - A) = \sec A$$

## **TRIGONOMETRIC IDENTITIES**

(a)  $\cos^2 A + \sin^2 A = 1$

(b)  $\sec^2 A - \tan^2 A = 1$

(c)  $\operatorname{cosec}^2 A - \cot^2 A = 1$

1. If  $\sin A = \frac{4}{5}$ , then find  $\cos A$  and  $\tan A$ .

2. If  $\operatorname{cosec} A = \frac{13}{5}$ , then find  $\cot A$ ,  $\sec A$  and  $\tan A$ .

3. If  $\cot \theta = \frac{7}{8}$ , evaluate

i)  $\frac{(1+\sin \theta)(1-\sin \theta)}{(1+\cos \theta)(1-\cos \theta)}$

ii)  $\cot^2 A$

4. If  $\sin(A-B) = \frac{1}{2}$ , and  $\cos(A+B) = \frac{1}{2}$ ,  $0 < (A+B) \leq 90^\circ$ , then find  $A$  and  $B$ .

5. Evaluate :  $2 \tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$

6. If  $\tan(A-B) = \sqrt{3}$ , and  $\tan(A+B) = \frac{1}{\sqrt{3}}$ ,  $0 < (A+B) \leq 90^\circ$ ,  $A > B$ , then find  $A$  and  $B$ .

7. If  $\sin 3A = \cos(A-26^\circ)$ , where  $3A$  is an acute angle. find the value of  $A$ .

8. Evaluate : i)  $\frac{\tan 65^\circ}{\cot 25^\circ}$  ii)  $\frac{\sin 18^\circ}{\cos 72^\circ}$

iii)  $\operatorname{cosec} 31^\circ - \sec 59^\circ$

9. Show that :  $\cos 36^\circ \cos 54^\circ - \sin 36^\circ \sin 54^\circ = 0$

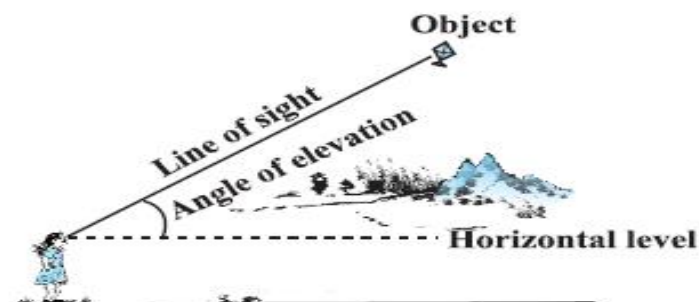
10. If  $\tan A = \cot B$ , prove that  $A+B = 90^\circ$

11. Prove that  $\sec A(1-\sin A)(\sec A + \tan A) = 1$ .

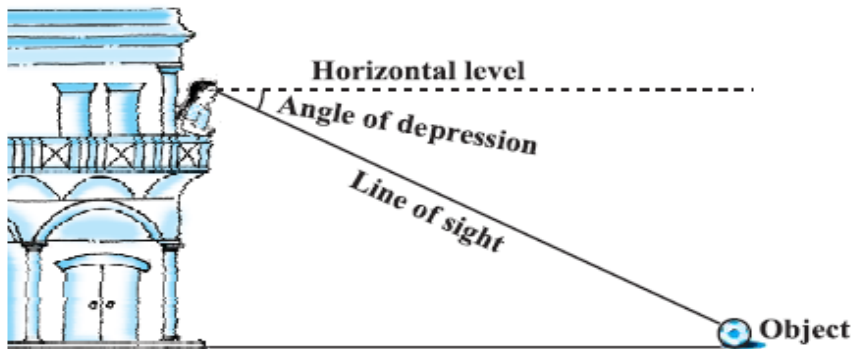
12. Evaluate:  $\frac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 73^\circ}$

13. Prove that :  $[\operatorname{cosec} \theta - \cot \theta]^2 = \frac{1-\cos \theta}{1+\cos \theta}$

## **Angle of Elevation**



## Angle of depression



**Line of sight** – It is the line drawn from the eye of an observer to the point in the object viewed by the observer.

**Angle of elevation** – It is the angle formed by the line of sight with the horizontal line when the point being viewed is above the horizontal level.

**Angle of depression** - It is the angle formed by the line of sight with the horizontal line when the point being viewed is below the horizontal level that is the case when we lower our head to look at the point being viewed.

1. The angle of elevation of the top of a tower from a point on the ground, which is 30 meter away from the foot of the tower, is  $30^\circ$ . Find the height of the tower.
2. The angle of elevation of the top of a water tank from a point on the ground, which is 50 meter away from the foot of the water tank, is  $45^\circ$ . Find the height of the Water tank.
3. A pole is 6 meter high casts a shadow  $2\sqrt{3}$  meter long on the ground. then find the sun's elevation?
4. The angle of depression of a car, standing on ground, from the top of a 75 meter high tower, is  $30^\circ$ . Find the distance of the car from the base of tower.
5. The Horizontal distance between two poles is 15 meter. The angle of depression of the top of first pole as seen from the top of second pole is  $30^\circ$ . If the height of second pole is 24 meter, find the height of first pole [ use  $\sqrt{3} = 1.732$  ].
6. The angle of elevation of top of building from the foot of tower is  $30^\circ$ . The angle of elevation of top of the tower from the foot of building is  $60^\circ$ . If the tower is 50 meter high, find the height of the building.
7. If the length of the shadow and the height of the tower are in the ratio 1: 1. then find the angle of elevation.
8. Find the angle of elevation of the tower's altitude, when the length of shadow equals to its height.
9. A ladder, leaning against a wall, makes a angle of  $60^\circ$  with the horizontal. if foot of the ladder is 2.5 meter away from the wall. Find the length of the ladder.

10. Find the angle of elevation of the sun, When the length of a tree is 3 times the height of the tree.
11. A man standing on the deck of a ship, which is 10 meter above the water level, observes the angle of elevation of the top of a hill as  $60^\circ$  and the angle of depression of the base of hill as  $30^\circ$ . find the distance of hill from the ship and the height of the hill.
12. If a tower 30 meter high, casts a shadow  $10\sqrt{3}$  meter long on ground, then what is the angle of elevation of the sun?
13. On a straight line passing through the foot of a tower, two points P and Q are at a distance of 4 m and 6 m from the foot respectively. If the angles of elevation from P and Q of the top of the tower are complementary, then find the height of the tower. [complementary angles are the angles whose sum is  $180^\circ$ ]
14. An electric pole is 10 m high. A steel wire tied to the top of the pole is affixed at a point to keep up right. if the wire makes angle of  $45^\circ$  with the horizontal through foot of the pole, Find the length of wire.
15. The angle of elevation of an aeroplane from a point on the ground is  $45^\circ$ . After flight for 15 seconds the elevation changes to  $30^\circ$ . if the aeroplane is flying at a height of 3000 m. find the speed of the aeroplane.
16. The angle of elevation of an aeroplane from a point on the ground is  $60^\circ$ . After flight of 15 seconds the elevation changes to  $30^\circ$ . if the aeroplane is flying at a constant height of  $1500\sqrt{3}$  m, find the speed of the plane in km/hour.

## **8. TOPIC: CIRCLE**

1. Theorem: The lengths of tangents drawn from an external point to a circle are equal.
2. Theorem: The tangent at any point of a circle is perpendicular to the radius through the point of contact.
3. A triangle PQ at a point P of a circle of radius 5cm meets a line through the centre O at a point Q so that  $OQ = 12$ cm find PQ
4. Prove that the tangents drawn at the ends of a diameter of a circle are parallel.
5. The length of a tangent from a point A at distance 5cm from the centre of the circle is 4cm, find the radius of the circle.
6. A quadrilateral ABCD is drawn to circumscribe a circle. Prove that  $AB + CD = AD + BC$ .
7. A triangle ABC is drawn to circumscribe a circle of radius 4cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8cm and 6cm respectively Find the sides AB and AC.

8. If TP and TQ are the two tangents to a circle with centre O so that  $\angle POQ = 110^\circ$  then find  $\angle PTQ$
9. Prove that in two concentric circles the chord of the larger circle, which touches the smaller circle is bisected at the point of contact.
10. Two concentric circles are of radii 5cm and 3cm. find the length of the chord of the larger circle which touches the smaller circle.
11. Two tangents PQ and PR are drawn from an external point to a circle with centre O. prove that QORP is a cyclic quadrilateral.
12. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre.
13. A circle is inscribed in  $\triangle ABC$  having sides  $AB=8\text{cm}$ ,  $BC=7\text{cm}$  and  $AC=5\text{cm}$ . find AD, BE and CF
14. The tangent at a point C of a circle and diameter AB when extended intersect at P. if  $\angle PCA = 110^\circ$ , find  $\angle CBA$
15. the incircle of a  $\triangle ABC$  touches the sides BC, CA and AB at D, E and F respectively If  $AB = AC$  prove that  $BD = CD$

## **9. CONSTRUCTION**


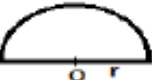
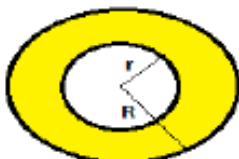


1. Construct the basic angles:-  $60^\circ$ ,  $30^\circ$ ,  $90^\circ$ ,  $45^\circ$  and  $75^\circ$ .
2. Draw a line segment of 7cm internally in the ratio 2:3
3. Draw a line segment  $AB = 9\text{cm}$  and divide it in the ratio 4:3
4. Draw a circle of radius 4cm Take a point P on it. Draw tangent to the given circle at P
5. Construct an isosceles triangle whose base 7.5cm and altitude is 4.2cm
6. Draw a line segment of length 7.6cm and divide it in the ratio 7:5 Measure the length of the smaller part.
7. Construct a triangle of sides 4cm, 5cm and 6cm and then triangle similar to it whose side are  $\frac{2}{3}$  of corresponding sides of the first triangle.
8. Draw a right triangle ABC in which  $\angle B = 90^\circ$   $AB=5\text{cm}$ ,  $BC=4\text{cm}$  then construct another triangle ABC whose sides are  $\frac{5}{3}$  times the corresponding sides of  $\triangle ABC$
9. Draw a line segment AB and locate a point C on AB such that  $AC = \frac{2}{7} AB$
10. Draw a triangle ABC with side  $BC=6\text{cm}$ ,  $AB=5\text{cm}$  and  $\angle ABC = 60^\circ$  Then construct a triangle whose sides are  $\frac{3}{4}$  of the corresponding sides of  $\triangle ABC$
11. Draw a circle of radius 3.5cm and construct a tangent to this circle making an angle of  $30^\circ$  with a line passing through the centre of the circle
12. Draw a pair of tangents to a circle of radius 6cm which are inclined to each other at the angle of  $60^\circ$ .
13. Draw a circle of radius 3cm From a point P outside the circle at a distance of 5cm from the centre of the circle, draw two tangents to the circle. Measure the length of these tangents.
14. Draw a circle with the help of a bangle. Find its centre.

15. Draw a circle with the help of a bangle. Take a point outside the circle. Construct the pair of tangents from this point to the circle.

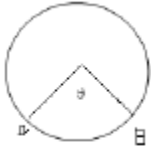
## 10. AREAS RELATED TO CIRCLES

### CONCEPTS

1. Circle: The set of points which are at a constant distance from a fixed point in a plane is called a circle.
2. Circumference: The perimeter of a circle is called its circumference.
3. Secant: A line which intersects a circle at two points is called secant of the circle.
4. Arc: A continuous piece of circle is called an arc of the circle.
5. Central angle: - An angle subtended by an arc at the center of a circle is called its central angle.
6. Semi-Circle: - A diameter divides a circle into two equal arcs. Each of these two arcs is called a semi-circle.
7. Segment: - A segment of a circle is the region bounded by an arc and a chord, of a circle.
8. Sector of a circle: The region enclosed by an arc of a circle and its two bounding radii is called a sector of the circle.
9. Quadrant: - One fourth of a circle/ circular disc is called a quadrant. The central angle of a quadrant is 90°.

S.N	NAME	FIGURE	PERIMETER	AREA
1.	Circle		$2\pi r$ or $\pi d$	$\pi r^2$
2.	Semi- circle		$\pi r + 2r$	$\frac{1}{2} \pi r^2$
3.	Ring (Shaded region)		$2\pi(r + R)$	$\pi(R^2 - r^2)$
4.	Sector of a circle		$l + 2r = \frac{\pi r \theta}{180^\circ} + 2r$	$\frac{\pi r^2 \theta}{360^\circ}$ or $\frac{1}{2} lr$
5.	Area of Segment of a circle		$\frac{\pi r \theta}{180^\circ} + 2r \sin \frac{\theta}{2}$	$\frac{\pi r^2 \theta}{360^\circ} - \frac{1}{2} r^2 \sin \theta$

a. Length of an arc  $AB = \frac{\theta}{360} 2\pi r$



b. Area of major segment = Area of a circle – Area of minor segment

### **CONCEPTUAL PROBLEMS**

1. If the perimeter of a circle is equal to that of square, then the ratio of their areas is

- i. 22/7
- ii. 14/11
- iii. 7/22
- iv. 11/14

2. The area of the square that can be inscribed in a circle of 8 cm is

- i. 256 cm<sup>2</sup>
- ii. 128cm<sup>2</sup>
- iii.  $64\sqrt{2}$ cm<sup>2</sup>
- iv. 64cm<sup>2</sup>

3. Area of a sector to circle of radius 36 cm is  $54\pi$  cm<sup>2</sup> . Find the length arc of the corresponding arc of the circle is

- i.  $6\pi$  cm
- ii.  $3\pi$  cm
- iii.  $5\pi$  cm
- iv.  $8\pi$  cm

4. A wheel has diameter 84 cm. The number of complete revolution it will take to cover 792 m is.

- i. 100
- ii. 150
- iii. 200
- iv. 300

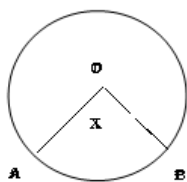
5. The length of an arc of a circle with radius 12cm is 10 cm. The central angle of this arc is.

- i. 1200
- ii. 60
- iii. 750
- iv. 1500

6. The area of a circle whose circumference  $\pi$  cm is

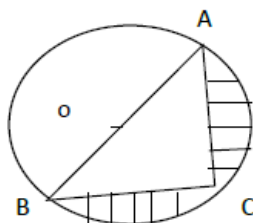
- i.  $11/2$  cm<sup>2</sup>
- ii.  $\pi/4$  cm<sup>2</sup>
- iii.  $\pi/2$  cm<sup>2</sup>
- iv. None of these

7. In figure 'o' is the centre of a circle. The area of sector OAPB is  $5/18$  of the area of the circle find x.

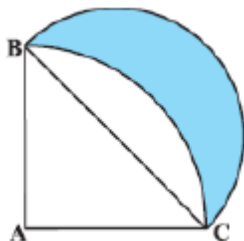


8. If the diameter of a semicircular protractor is 14 cm, then find its perimeter.
9. The diameter of a cycle wheel is 21cm. How many revolutions will it make to travel 1.98km?
10. The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.

11. Find the area of the shaded region in the figure if  $AC=24$  cm , $BC=10$  cm and O is the center of the circle.



12. ABC is a quadrant of circle of radius 14 cm and a semi-circle is drawn with BC as diameter. Find the area of shaded region.



## **11. SURFACE AREA AND VOLUMES**

KEY CONCEPTS:

### **1. CUBOID :**

- (I) TOTAL SURFACE AREA OF A CUBOID :  $2( LB + BH + HL )$
- (II) Volume of a cuboid =  $L \times B \times H$  sq units
- (III) Diagonal of a cuboid =  $\sqrt{L^2+B^2+H^2}$  units

### **2. CUBE :**

- (I) Total Surface Area of a Cube =  $6a^2$  sq units
- (II) Volume of the Cube =  $a^3$  cubic units
- (III) Diagonal of a Cube =  $\sqrt{3}a$

### 3. Right Circular Cylinder :

- (I) Curved Surface Area =  $2\pi rh$
- (II) Total Surface Area =  $2\pi r (h + r)$
- (III) Volume =  $\pi r^2 h$

### 4. Right Circular Hollow Cylinder :

- (I) Area of each end =  $\pi R^2 - \pi r^2$  [ R and r be the external radius and internal radius ]
- (II) Curved Surface Area of Hollow Cylinder =  $2\pi h (R + r)$
- (III) Total Surface Area =  $\pi (R + r) [ 2h + R - r ]$
- (IV) Volume of material =  $\pi h (R^2 - r^2)$

### 5. Sphere :

- (I) Surface Area =  $4\pi r^2$
- (II) Volume =  $\frac{4}{3} \pi r^3$

### 6. Hemisphere(SOLID) :

- (I) Curved Surface Area =  $2\pi r^2$
- (II) Total Surface Area =  $3\pi r^2$
- (III) Volume =  $\frac{2}{3} \pi r^3$

### 7. Right Circular Cone :

- (I) Curved Surface Area =  $\pi rl$  [ l = Slant Height ]
- (II) Total Surface Area =  $\pi r (L + r)$  sq units
- (III) Volume =  $\frac{1}{3} \pi r^2 h$

### 8. Frustum of a Cone :

- (I) Volume of a Frustum of a Cone =  $\frac{\pi h}{3} (R^2 + r^2 + Rr)$  [ R = radius of base, r = radius of frustum ]
- (II) Lateral Surface Area of the Frustum of a cone =  $\pi L (R + r)$  [ where  $L^2 = h^2 + (R - r)^2$  ]
- (III) Total Surface Area of the Frustum of the cone =  $\pi [ R^2 + r^2 + L (R + r) ]$  sq units

## CONCEPTUAL PROBLEMS

1. The Surface Area of a Sphere is 616 cm<sup>2</sup>. Find its radius.
2. The slant height of the frustum of a cone is 5 cm. if the difference between the radii of its two circular ends is 4cm, write height of the frustum.
3. A cylinder and a cone area of the same base radius and of the same height. Find the ratio of the cylinder to that of the cone.
4. Two cones have their heights in the ratio 1:3 and radii 3:1. What is the ratio of their volumes?



5. The radii of two cones are in the ratio 2:1 and their volumes are equal. What is the ratio their heights?
6. The diameter of a sphere is 6 cm. it is melted and drawn into a wire of diameter 2mm. Find the length of the wire.
7. Find the curved surface area of a right circular cone of height 15cm and base diameter is 16 cm.
8. Find the maximum volume of a cone that can be out of a solid hemisphere of radius r.
9. The diameter of the ends of a frustum of a cone are 32cm and 20 cm. If its slant height is 10 cm. Find the lateral surface area.
10. Find the curved surface area of a right circular cone of height 15cm and base diameter is 16 cm
11. A cone of height h and a sphere have the same radii r and same volume, then find r : h.
12. Metallic sphere of radii 6cm, 8cm and 10cm respectively, are melted to form a single solid sphere. Find the radius of the resulting sphere
13. A circus tent is cylindrical up to a height of 3m and conical above it. If the diameter of the base is 105m and the slant height of the conical part is 53m. Find the total canvas used in making the tent.
14. The largest sphere is curved out of a cube of a side 7cm. Find the volume of the sphere
15. Two cubes of volume 64cm<sup>3</sup> are joined end to end. Find the volume of the sphere.

## **12.STATISTICS**

### GIST OF LESSON:

1. Ungrouped and Grouped frequency distribution.
2. Measures of central tendency – MEAN . MODE. And MEDIAN.
3. Mean of ungrouped data = Sum of all the observations / Total no. of observations
4. If  $x_1, x_2, x_3, \dots, x_n$  are the observations with frequencies  $f_1, f_2, f_3, \dots, f_n$  then Mean of the data is given by ,

$$\text{Mean} = \frac{f_1x_1 + f_2x_2 + \dots + f_nx_n}{f_1 + f_2 + \dots + f_n} = \frac{\sum f_i x_i}{\sum f_i}$$

Where  $\sum$  is a Greek symbol used for summation .

Mean of grouped Data : ( a ) Direct method -  $\text{Mean} = \frac{\sum f_i x_i}{\sum f_i}$  , where  $x_i$  is the class mark given by,  $x_i = \frac{\text{upper limit} + \text{lower limit}}{2}$

( b ) Assumed mean method :  $\text{Mean} = a + \frac{\sum f_i d_i}{\sum f_i}$  where  $d_i = x_i - a$  and a = assumed mean.

5. MODE: An observation with highest frequency is the mode or an observation which occurs maximum number of times is the mode.

6. Mode of grouped data : in a grouped frequency distribution , it is not possible to find the mode by looking at the frequencies. Here we can locate a class with the highest frequency, called the modal class. The mode is a value inside the modal class and is given by the formula

$$Mode = l + h \left( \frac{f_m - f_1}{2f_m - f_1 - f_2} \right)$$

Where,

$l$  = Lower Boundary of modal class

$h$  = size of modal class

$f_m$  = Frequency corresponding to modal class

$f_1$  = Frequency preceding to modal class

$f_2$  = Frequency proceeding to modal class

7. Cumulative frequency:

Class Limits	Frequency	Cumulative Frequency
5-10	1	1
10-15	2	3
15-20	4	7
20-25	0	7
25-30	3	10
30-35	5	15
35-40	6	21

7. MEDIAN : Median is the value of middle most observation.

( a ) For finding the median of ungrouped data , we first arrange the data in ascending order , then if  $n$  is odd, the median is the  $( n+ 1 / 2 )$  th observation and if  $n$  is even then median is the average of  $n/2$  th and  $( n/2 + 1)$ th observation.

( b) For grouped data Median is given by :

$$Median = l + \frac{h}{f} \left( \frac{N}{2} - c \right)$$

Where:

$l$  = lower class boundary of the median class

$h$  = Size of the median class interval

$f$  = Frequency corresponding to the median class

$N$  = Total number of observations i.e. sum of the frequencies

$c$  = Cumulative frequency preceding median class.

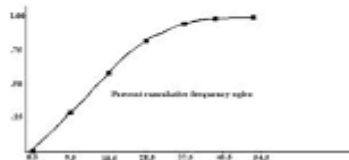
Median class : To find this class, we find the cumulative frequencies of all the classes and  $n/2$  . Now we locate the class whose cumulative frequency is greater than ( and nearest to )  $n/2$  . This is called the median class.

Empirical formula :  $3\text{Median} = \text{Mode} + 2\text{Mean}$

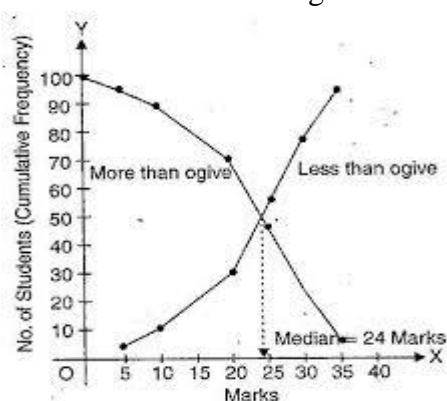
## 9. Cumulative frequency curve or an ogive :

### Ogive

- The ogive is a graph that represents the cumulative frequencies for the classes in a frequency distribution.



The median of grouped data can be obtained graphically as the x- coordinate of the point of intersection of the two ogives less than type and more than type.



### QUESTIONS

- Find the mean of first five whole numbers .
- How will you find the class mark (  $x_i$  ) ? .
- In a mathematics given to 15 students , the following marks were recorded( out of 100 ) :

41,39,48,52,46,62,54,40,96,52,98,40,42,52,60 .

- Find the mean , median and mode of this data .

- Find the mean of the following by Direct method :

Marks	0 - 10	10- 20	20- 30	30- 40	40- 50	50-60
No. of students	3	9	17	12	9	10

- Find the mode of the following : 14,25,14,28,18,17,18,14,23,22,14,18 .
- Find the median using empirical formula when it is given that mode and mean are 8 and 9 respectively .
- Write the formula for finding the mode of grouped data.
- What is modal class ?
- Find the mode of the following:

Length (in mm)	Number or leaves $f_i$
118 - 126	3
127 - 135	5
136 - 144	9
145 - 153	12
154 - 162	5
163 - 171	4
172 - 180	2

11.

The following table shows marks secured by 140 students in an examination:

Marks	0-10	10-20	20-30	30-40	40-50
Number of students	20	24	40	36	20

Calculate mean marks by using all the three methods, i.e., direct method, assumed mean method and step-deviation method.

12. Find the Median:

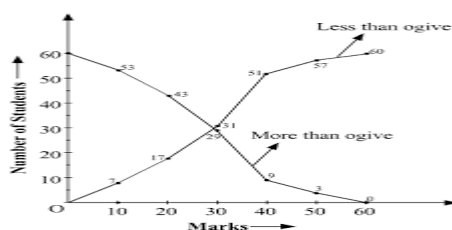
Length (in mm)	Number or leaves $f_i$
118 - 126	3
127 - 135	5
136 - 144	9
145 - 153	12
154 - 162	5
163 - 171	4
172 - 180	2

13. If the mean of the following data is 21.5 then find the value of k :

$X_i$	5	15	25	35	45
$F_i$	6	4	3	k	2

14. Which measure of central tendency can be obtained from cumulative frequency graph or ogive ?

15. Find the median from the below graph.



16. During the medical check-up of 35 students of a class, their weights were recorded as follows: Draw a less than type ogive for the given data. Hence obtain the median weight from the graph and verify the result by using formula.

Weight (in Kg)	Number of Students
Less than 38	0
Less than 40	3
Less than 42	5
Less than 44	9
Less than 46	14
Less than 48	28
Less than 50	32
Less than 52	35